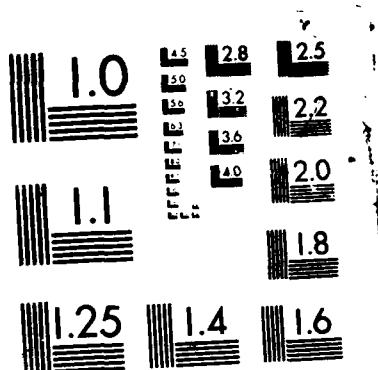


AD-A186 248 RELIABILITY AND SURVIVABILITY OF COMMUNICATION NETWORKS 1/1
(U) ALABAMA UNIV IN HUNTSVILLE P J SLATER ET AL
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Annual Report

For

RELIABILITY AND SURVIVABILITY OF COMMUNICATION NETWORKS

Contract No. N00014-86-K-0745

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RE: Annual Letter Report
Principal Investigator: Peter J. Slater
Contract Name: FONR Comm Networks
Contract No.: N000K4-86-K-0745
Account No.: 5-31504
Co-investigators: Ashok T. Amin
Kyle T. Siegrist
Period Covered: June, 1986- May, 1987

Publications:

1. On the expected number of pairs of connected nodes: pair-connected reliability, to appear in *Computers and Mathematics with Applications*.
2. Exact formulas for reliability measures for various classes of graphs, to appear in *Proc. 18'th Southeastern Conf. on Combinatorics, Graph Theory and Computing*.
3. Pair-connected reliability of a tree and its distance degree sequences, to appear in *Proc. 18'th Southeastern Conf. on Combinatorics, Graph Theory and Combinatorics*.

Presentations:

1. Dr. Slater presented 1 (above) at the Computer Networks Symposium, Las Cruces, NM, November, 1986.
2. Dr. Siegrist presented 2 (above) at the 18'th Southeastern Conference, February, 1987.
3. Dr. Amin presented 3 (above) at the 18'th Southeastern Conference, February, 1987.
4. Dr. Siegrist presented an invited lecture, "The expected number of pairs of connected vertices in probabilistic graphs," at the Georgia Institute of Technology, March, 1987.



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5..Dr. Slater presented an invited lecture, "The pair-connected measure of network reliability," at the Graphs and Algorithms Conf., Boulder, CO, June, 1987.

Summary report:

We assume that we have an (n,m) -graph G , a graph with n vertices and m edges, representing a communications network with each vertex representing a processor and each edge a communications link. We have begun our work by concentrating on the cutting number/pair-connected reliability measure and have initially assumed that each vertex will always remain operable and that each edge has the same probability of failure q . Thus, $p = 1 - q$ is the probability that any given edge is operable.

A standard measure of system reliability is the probability that the graph remains connected. This is denoted by $P(G;q)$, and the probability that a specified pair s and t of vertices is connected is denoted by $P_{s,t}(G;q)$. The cutting number/pair-connected measure refines the connectivity reliability measure by considering the varying effects of different cutsets. For example, edge cutsets $\{a,b\}$ and $\{c,d\}$ in graph G_1 produce five and nine pairs of disconnected vertices, respectively. We let $CN(G;q)$ denote the expected number of pairs of disconnected vertices, and $PC(G;q) = \binom{n}{2} - CN(G;q)$ is the expected number of pairs of vertices that are connected.

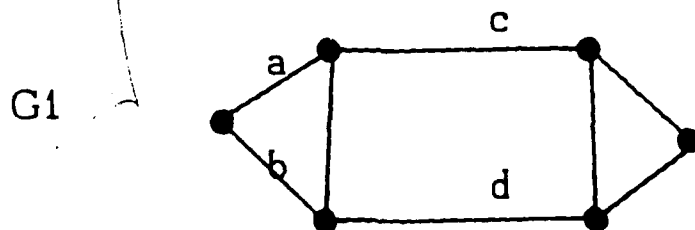


Figure 1. A $(6,8)$ -graph G_1 .

An appropriate observation at this point is the following. Many articles in the literature are concerned with the design of optimal graphs for small values of the component failure probability q . This is reasonable when we note that communications links are well enough designed so as to be highly reliable and when we are simply interested in one processor being able to access another without significant concern about the speed of access. Assume, however, that each communication link has a capacity, and we think of q as the probability that a communication link is at capacity at any given instant. In a "well-utilized" system q will be large. In particular, we must consider values of q over the whole interval $(0,1)$ when designing an optimally reliable (n,m) -graph.

While it has become necessary in our studies to consider both disconnected and connected graphs, we assume in this report that the entire communications network is connected if no edges have failed. Hence $m \geq n-1$ and the first case to consider is that with $m = n-1$, namely, the n -vertex trees. The simple connectivity reliability measure does not distinguish between two $(n,n-1)$ -trees because with any edge failure a tree becomes disconnected. However, there exist trees T_1 and T_2 such that $PC(T_1;q) - PC(T_2;q)$ takes on positive and negative values for different values of q . That is, each tree is more reliable than the other for certain intervals of $q \in (0,1)$. Calling an (n,m) -graph G uniformly optimal if $PC(G;q) \geq PC(H;q)$ ($0 < q < 1$) for every (n,m) -graph H , Boesch has conjectured that for the measure of simple graph connectivity there always exists a uniformly optimal (n,m) -graph. To the contrary, we have shown that in general there does not exist a uniformly optimal pair-connected (n,m) -graph. An exception is that the

star is a uniformly optimal tree (and the path P_n on n vertices is "uniformly worst"). For an arbitrary connected graph G let $(D_1, D_2, \dots, D_{n-1})$ be its distance distribution, so that D_i is the number of pairs of vertices at distance i . For an arbitrary graph its distance distribution provides a lower bound on the expected number of pairs of connected vertices, while for a tree T the distance distribution completely determines $PC(T;q)$. Specifically,

$$PC(T;q) = \sum_{i=1}^{n-1} D_i p^i.$$

In publication [3] we show that graph realization of (the closely related concept of) distance degree sequences is one of the very few problems which are NP-complete when restricted to trees.

$PC(T;q)$ is easy to compute for a tree T , as shown in [1], however, as independently shown by C. Colbourn, the computation of $PC(G;q)$ is NP-complete even when restricted to planar graphs of maximum degree four. In general we can write $PC(G;q)$ as

$$PC(G;q) = \sum_{i=1}^m A_i p^i.$$

While we have produced programs to compute $PC(G;q)$, because of the expected exponential run time of any such algorithm, we are currently completing development of a computation/simulation package. This package will allow exact computation of the coefficients of the reliability polynomials $P(G;q)$, $P_{s,t}(G;q)$ and $PC(G;q)$ for graphs of small size, and will permit computation of approximate values of the reliability measures, for a given value of q , for graphs of large size.

The two principal results in a paper currently being written are, first, the nonexistence in general of uniformly optimal (n,m) -graphs for pair-connected reliability, and, second, the coefficient A_i in $PC(G;q)$ is

determined by an easily described subset of the collection of subgraphs of G on i edges. In particular, this implies the existence of a polynomial time algorithm to determine A_1, A_2, \dots, A_k for a fixed k . Further the associated theory is allowing us to design (n,m) -graphs which are optimal for small p (large q).

Many problems related to pair-connected reliability are under study, including providing exact formulas for $PC(G;q)$ for various classes of graphs G . One particularly important problem is the design of (n,m) -graphs which are optimal for large values of p (small q).

$PC(G;q)$ is the expected value of the random variable which gives the number of pairs of connected vertices in the probabilistic graph G with edge failure probability q . As such, it is but one of many possible measures of the center of the distribution of this random variable. The distribution itself is hopelessly complicated to compute, even for the simplest graphs. Thus, it is fundamentally important to obtain qualitative information about the shape of the distribution and asymptotic information about the limit of the distribution (when appropriately scaled) as the size of the graph grows large. Results of this latter type are termed *central limit theorems* in the probability literature. So far, we have established central limit theorems for the path and the star which show that the number of pairs of connected vertices has an asymptotically normal distribution for either of these types of graphs. These results suggest that the number of pairs of connected vertices may be approximately normally distributed for any large tree and indeed perhaps for any large graph. We are vigorously pursuing this line of investigation.

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